GENERAL

anova reml stats

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Understanding ANOVA and REML in Modern Statistical Practice

When researchers analyze data that come from experiments, surveys, or observational studies, one of the most common questions is whether a set of treatments or factors truly affects an outcome of interest. The classic tool to answer this question is **Analysis of Variance**, or **ANOVA**. While the basic ANOVA model assumes that all sources of variation are fixed, many real world datasets contain random effects—variation that arises from sampling units, blocks, or subjects. In such cases, the traditional ANOVA framework is insufficient, and statisticians turn to mixed effects modeling. A key estimation technique in mixed effects models is **Restricted Maximum Likelihood** (REML). This article delves into the theory, practice, and practicalities of combining ANOVA with REML in statistical analyses, offering a comprehensive guide for researchers, analysts, and students alike.

What Is ANOVA? A Quick Refresher

ANOVA was first developed in the early twentieth century to compare means across multiple groups while controlling for the overall variance in the data. At its core, ANOVA partitions the total variation observed in a response variable into components attributable to different sources:

- Between group variation: Differences that can be explained by the factors or treatments.
- Within group variation: Random error or residual variation that remains after accounting for the factors.

The classic one way ANOVA tests the null hypothesis that all group means are equal. If the between group variation is significantly larger than the within group variation, the null hypothesis is rejected, suggesting that at least one treatment differs.

In the simplest form, the ANOVA model can be written as:

Y_{ij} = ;Â ² Ä_i + ;SÇ7V#æ–£Â÷7V#à

where *Y_{ij}* is the response for the *j*th observation in the *i*th group, *;À* is the overall mean, *<CÇ7V#æ“Â÷7V#à* represents the treatment effect (fixed effect), and *;SÇ7V#æ–£Â÷7V#à* is a random error term assumed to be normally distributed with mean zero and variance <;"à

Why Random Effects Matter

In many experimental designs, researchers encounter sources of variability that are not fixed. For example:

- Students nested within classrooms.
- Patients nested within hospitals.
- Repeated measurements on the same subject.

These are **random effects** because they represent a random sample from a larger population of possible units. Ignoring random effects can lead to inflated Type I error rates and biased estimates of fixed effects. Mixed effects models, which combine both fixed and random effects, are the standard approach to handle such complexity.

Introducing Restricted Maximum Likelihood (REML)

When estimating the parameters of a mixed effects model, two primary approaches exist:

1. Maximum Likelihood (ML): Estimates both fixed and random effects by maximizing the likelihood of the observed data.
2. Restricted (or Residual) Maximum Likelihood (REML): Focuses on estimating variance components (random effects) by maximizing the likelihood of a set of linear combinations of the data that are independent of the fixed effects.

REML is particularly advantageous because it reduces bias in the estimation of variance components, especially in small samples or unbalanced designs. While ML can produce biased estimates of variance components, REML corrects for this by accounting for the loss of degrees of freedom associated with estimating fixed effects.

Comparing ANOVA and REML: When to Use Which?

Below is a concise comparison to help decide which approach is appropriate for a given dataset:

- ANOVA (Fixed Effects Only)\n\nAssumes all sources of variation are fixed.
- Suitable for balanced designs with no random effects.
- Uses F-tests for hypothesis testing.
- Does not provide estimates for random variance components.

- Handles both fixed and random effects.
- Ideal for unbalanced designs, nested or crossed random factors.
- Provides unbiased estimates of variance components.
- Allows for more flexible hypothesis testing (e.g., likelihood ratio tests, Wald tests).

In practice, researchers often start with a traditional ANOVA to get a quick sense of group differences, then move to a mixed effects model with REML for more nuanced analysis.

Linear Mixed Models: The General Framework

A linear mixed model can be expressed as:

Y = X;" ² ¦" ² µ

where:

- Y is the vector of observed responses.
- X is the design matrix for fixed effects.
- ;" —2 F†R `ector of fixed effect coefficients.
- Z is the design matrix for random effects.
- b is the vector of random effect coefficients, assumed to follow a multivariate normal distribution with mean zero and covariance matrix G.
- ;R —2 F†R &W6–GV Â W'&÷" `ector with covariance matrix R.

REML estimation focuses on the distribution of **y** after eliminating the fixed effects, yielding an objective function that depends only on the variance components in **G** and **R**.

Variance Component Estimation with REML

Suppose we have a random intercept model:

Y_{ij} = ;Â ² #Ç7V#æ“Â÷7V#â ² µ_{ij}

where *b_i* ~ N(0, <6+"' æ‚BÇ7V#æ–£Â÷7M(0, <;"'à REML estimates <6+ and <; by maximizing the likelihood of the residuals after accounting for ;Âà This often yields more reliable variance component estimates than ML, especially when the number of groups (i) is small relative to the number of observations.

Implementing ANOVA with REML in Popular Software

Below is a quick guide to performing mixed effects ANOVA using REML in several widely used statistical packages.

R: The lme4 and nlme Packages

The **lme4**package is the most popular for fitting linear mixed models. It uses REML by default when fitting models with random effects.

library(lme4)\nmodel <- lmer(response ~ treatment + (1 | subject), data = mydata)\nsummary(model)\nanova(model)\n\nThe **nlme**package also supports REML and offers more flexible covariance structures.

library(nlme)\nmodel <- lme(response ~ treatment, random = ~1 | subject, data = mydata)\nsummary(model)\nanova(model)\n\n\nSAS: PROC MIXED

proc mixed data=mydata;\n class treatment subject;\n model response = treatment / solution;\n random intercept / subject=subject;\n run;\n\nBy default, PROC MIXED uses REML for estimating variance components unless the **METHOD=ML** option is explicitly specified.

SPSS: MIXED Procedure

1. Analyze > Mixed Models > Linear.
2. Define fixed effects (e.g., treatment) and random effects (e.g., subject).
3. Choose REML under the Estimation method.

Stata: Mixed Command

mixed response treatment || subject:\n\nStata's **mixed** command automatically uses REML unless the **ml**option is specified.

Case Study: An Unbalanced Agricultural Experiment

Imagine a study where three fertilizer types (A, B, C) are applied to plots of land, but due to logistical constraints, the number of plots per fertilizer varies: 10 plots for A, 8 for B, and 12 for C. Each plot yields a crop weight measurement. A traditional one way ANOVA might still be performed, but it would ignore the imbalance and potentially yield misleading results.

model <- lmer(crop_weight ~ fertilizer + (1 | plot), data = agri_data)\n\n\n**Step 2: Examine Variance Components**\nThe REML output provides estimates of the residual variance and the plot variance. If the plot variance is substantial, it indicates that plot differences significantly contribute to yield variability.

The **anova**function or likelihood ratio tests can assess whether fertilizer has a statistically significant effect, controlling for plot variability.

By employing REML, the analysis appropriately adjusts for the unbalanced design, leading to more accurate inference about the fertilizer effects.

Statistical Inference: F Tests, Likelihood Ratio Tests, and Confidence

Intervals

In mixed effects models, hypothesis testing can be carried out through several approaches:

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