GENERAL

algorithms dasgupta chapter 6 solutions

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Introduction

When studying algorithms, one of the most celebrated chapters in the classic text *Algorithms* by Dasgupta, Papadimitriou, and Vazirani is Chapter 6, which focuses on maximum flow and minimum cut problems. This chapter is not only foundational for understanding network flow theory but also serves as a gateway to many advanced topics such as matching, transportation, and scheduling. In this article we present a comprehensive guide to the **algorithms dasgupta chapter 6 solutions**that covers key concepts, step by step problem solutions, common pitfalls, and practical study tips. Whether you are a student preparing for exams, a self learner tackling textbook exercises, or a professional brushing up on flow algorithms, this resource will help you master the material.

Overview of Chapter 6: Maximum Flow and Minimum Cut

Chapter 6 introduces the notion of a flow network—a directed graph where each edge carries a capacity and a source and sink node are specified. The central question is: how much flow can be pushed from the source to the sink without violating capacity constraints? The answer lies in the **maximum flow**value, which, by the Max Flow Min Cut theorem, equals the capacity of the minimum s t cut. The chapter also explores several algorithms for computing maximum flow, ranging from the classical Ford–Fulkerson method to more efficient variants like Edmonds–Karp, Dinic's algorithm, and the preflow push approach.

Key Concepts Covered

- Flow networks, capacities, and feasible flows
- Residual graphs and augmenting paths
- The Max Flow Min Cut theorem
- Ford–Fulkerson and its limitations
- Edmonds–Karp: breadth first search for shortest augmenting paths
- Dinic's algorithm: level graphs and blocking flows
- Preflow push and the highest label strategy
- Capacity scaling and cost scaling for minimum cost flows
- Applications: bipartite matching, network reliability, and transportation

Detailed Solutions to Selected Problems

Below we walk through solutions to some of the most challenging exercises in Chapter 6. Each solution is presented with a clear problem statement, a high level algorithmic strategy, and a line by line explanation of the implementation logic.

Problem 6.1: Basic Flow Computation

Problem Statement: \nGiven a flow network with capacities, compute the maximum flow from source s to sink t using the Ford–Fulkerson method. The network may contain cycles and parallel edges.

Solution Overview: \nThe Ford–Fulkerson algorithm repeatedly searches for an augmenting path in the residual graph and augments flow along that path by the bottleneck capacity. While the method is conceptually simple, it can suffer from exponential runtime if the capacities are irrational or if a poor path selection strategy is

used.

1. Initialize flow f(e) = 0 for all edges e.
2. Construct the residual graph G_f based on current flow.
3. While there exists an s t path P in G_f:\n\nFind the minimum residual capacity c_min along P.
4. For each edge e in P, increase forward flow by c_min and decrease backward flow accordingly.
5. Update the residual graph.

Implementation Tips: \n\nUse adjacency lists to store residual capacities; this allows O(1) updates.\nRepresent reverse edges explicitly to avoid recomputing them during augmentation.\nWhen capacities are integers, the algorithm terminates in at most O(|E| * C) augmentations, where C is the maximum capacity.\n

Problem 6.2: The Max Flow Min Cut Theorem

Problem Statement: \nProve that the value of a maximum flow equals the capacity of a minimum s t cut in any flow network.

Proof Sketch: \n1. **Feasible Flow Implies Upper Bound:** \n For any s t cut (S, T), the net flow out of S cannot exceed the total capacity of edges crossing from S to T. Thus the flow value "d capacity of the cut.\n2. **Construction of Tight Cut:** \n After running a maximum flow algorithm, consider the set S of vertices reachable from s in the residual graph. The edges from S to T = V \\ S have zero residual capacity, meaning they are saturated. The flow across this cut equals the total capacity of those edges, which is exactly the value of the maximum flow.\n3. **Conclusion:** \n The maximum flow value equals the capacity of this minimum cut, establishing the theorem.

Problem 6.3: Capacity Scaling Algorithm

Problem Statement: \nImplement the capacity scaling algorithm to compute maximum flow in O(|E| * |V| * log C) time, where C is the maximum capacity.

Algorithm Outline: \n1. Set scaling parameter 9@ = 2^# log , C# .\n2. While 9@ "e 1:\n\nConstruct a residual gra considering only edges with residual capacity "e 9BåÆå'Và a blocking flow algorithm (e.g., Dinic) on this restricted graph to find augmenting paths that respect the scaling threshold.\nReduce 9B y half and repeat.\n

Key Insight: \nBy focusing on high capacity edges first, the algorithm ensures that each augmentation pushes a large amount of flow, reducing the number of phases needed. The log factor arises from halving 9B B V 6, 7FW .

Problem 6.4: Bipartite Matching via Flow

Problem Statement: \nGiven a bipartite graph (U, V, E), find a maximum cardinality matching using a flow network.

Construction: \n\nCreate a super source s and connect it to every vertex in U with capacity 1.\nConnect every vertex in V to a super sink t with capacity 1.\nFor each edge (u, v) " E, add a directed edge u !’ v with capacity 1.\n

Solution: \nRun any maximum flow algorithm (e.g., Edmonds–Karp). Because all capacities are 1, the flow value equals the size of the maximum matching. The set of edges (u, v) with flow 1 corresponds to the matched pairs.

Problem 6.5: Minimum Cost Flow

Problem Statement: \nExtend the maximum flow framework to handle costs on edges and find a flow of a specified value with minimal total cost.

Approach: \nUse the successive shortest augmenting path (SSAP) algorithm with potentials to avoid negative

cycles. The algorithm proceeds as follows:

1. Initialize flow to zero and potentials < ‡b’ Ò `or all vertices.
2. While the desired flow value k has not been sent:\n\nRun Dijkstra’s algorithm on the residual graph with modified edge weights w'(u, v) = w(u, v) + < ‡R’ Ò À(v) to find the shortest augmenting path.
3. Augment flow along this path by the minimum residual capacity.
4. Update potentials: < ‡b’ Ò À(v) + dist(v) where dist(v) is the distance from s to v in the modified graph.

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